RESEARCH OF NONSTATIONARY PLANE-PARALLEL PRESSURE FLOW OF VISCOUS LIQUID UNDER FIXED DIFFERENTIAL PRESSURE

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Nonstationary plane-parallel pressure flow of viscous liquid under the conditions of fixed differential pressure is reviewed.
Regularities of nonstationary hydromechanic parameter change of the plane-parallel pressure flow for the presence of fixed differential pressure are obtained. These regularities under the presence of fixed differential pressure allow to determine the character of ongoing processes.
It is obvious from the obtained diagrams that in low Reynolds number, transient process strives for the stationary mode, and in their large numbers – nonstationary mode maintains its condition for a long time.

Key words: *Plane-parallel nonstationary flow, instantaneous velocity, coefficient, viscous liquid, differential pressure.*

Regularities of change of hydromechanic parameters of nonstationary plane-parallel pressure flow of viscous liquid under arbitrary differential pressure and initial velocity distribution [1] were obtained. Based on common problem solutions [1], we will obtain the regularities of change of hydromechanic parameters of nonstationary plane-parallel pressure flow under fixed differential pressure.

Let viscous liquid at the beginning of nonstationary motion be at a rest, and in t=0, the fixed differential pressure affects liquid. In such case, initial and boundary conditions of the problem will be:

$$\varphi(y) = 0\,; \tag{1}$$

$$f(t) = -\frac{\partial P}{\partial x} = \frac{h}{u_\infty^2}\cdot\frac{P_1 - P_2}{\rho l} = P = \text{const}\,. \tag{2}$$

In these function values $\varphi(y)$ and $f(t)$, will calculate the value of coefficient $C_k(t)$ and function $F_k(t)$ [1]. Will obtain

$$C_k(t) = 0:$$

$$F_k(t) = \int_0^t P\cdot\exp\left(\frac{\pi^2(2k-1)^2}{4\,\text{Re}}u\right)du = \frac{4\,\text{Re}\,P}{\pi^2(2k-1)^2}\cdot\left(\exp\left(\frac{\pi^2(2k-1)^2}{4\,\text{Re}}t\right)-1\right)\,. \tag{3}$$

Putting the value of the function $C_k(t)$ and $F_k(t)$ into the equation (37) [1], will get the regulatory of the velocity change of plane-parallel pressure flow of viscous liquid when fixed differential pressure affects liquid at a rest.

$$u_x(y,t) = \frac{16\,\text{Re}\,P}{\pi^3}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)^3}\left(1-\exp\left(-\frac{\pi^2(2k-1)^2}{4\,\text{Re}}t\right)\right)\cos\left[\pi\frac{(2k-1)}{2}y\right]. \tag{4}$$

Taking into the account that $u_{max} = \dfrac{\text{Re}\,P}{2}$ last equality can be written in form of:

$$\frac{u_x(y,t)}{u_{max}} = \frac{32}{\pi^3}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)^3}\left(1-\exp\left(-\frac{\pi^2(2k-1)^2}{4\,\mathrm{Re}}t\right)\right)\cos\left[\pi\frac{(2k-1)}{2}y\right]. \tag{5}$$

Whereas,

$$\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)^3}\cdot\cos\left[\pi\frac{(2k-1)}{2}y\right] = \frac{\pi^3}{32}\left(1-y^2\right), \tag{6}$$

Therefore, the equality (5) will have the following form:

$$\frac{u_x(y,t)}{u_{max}} = \left(1-y^2\right) - \frac{32}{\pi^3}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)^3}\cdot\cos\left[\pi\frac{(2k-1)}{2}y\right]\exp\left(-\frac{\pi^2(2k-1)^2}{4\,\mathrm{Re}}t\right). \tag{7}$$

From the last equation will obtain

$$u_x(y,t) = U_{max}\left(1-y^2\right) - \frac{4\,\mathrm{Re}\cdot P}{\pi^3}\cdot\sum_{k=1}^{\infty}\frac{\cos\left[\pi\frac{(2k-1)}{2}y\right]}{(2k-1)^3}\cdot\exp\left(-\frac{\pi^2(2k-1)^2}{4\,\mathrm{Re}}t\right). \tag{8}$$

Will determine an average velocity of the effective cross-section from equations (40) [1] and (4):

$$V(t) = -\frac{64P\,\mathrm{Re}}{\pi^4}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^4}\left(\exp\left(-\frac{\pi^2(2k-1)^2}{4\,\mathrm{Re}}t\right)-1\right). \tag{9}$$

Shear stresses between the layers of accelerated liquid based on (38) [1] and (8) would be

$$\tau = \mu\frac{8P\,\mathrm{Re}}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}\left(1-\exp\left(-\frac{\pi^2(2k-1)^2}{4\,\mathrm{Re}}t\right)\right)\cdot\sin\left[\pi\frac{(2k-1)}{2}y\right]. \tag{10}$$

Momentum coefficients are determined under the formula (48) [1]

$$\begin{aligned}\beta &= \frac{8P^2\,\mathrm{Re}^2}{\pi^6V^2}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^6}\left(1-2\exp\left(-\frac{\pi^2(2k-1)^2}{4\,\mathrm{Re}}t\right)+\exp\left(-\frac{2\pi^2(2k-1)^2}{4\,\mathrm{Re}}t\right)\right) = \\ &= \frac{8P^2\,\mathrm{Re}^2}{\pi^6V^2}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^6}\left(1-\exp\left(-\frac{\pi^2(2k-1)^2}{4\,\mathrm{Re}}t\right)\right)^2, \qquad 0\le t<\infty.\end{aligned} \tag{11}$$

Coefficients of nonuniform distribution of the velocities would be

$$\alpha = \frac{\int_{-1}^{+1}\left(\frac{32}{\pi^3}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{(2k-1)^3}\left(1-\exp\left(-\frac{\pi^2(2k-1)^2}{4\,\mathrm{Re}}t\right)\right)\cos\left[\pi\frac{(2k-1)}{2}y\right]\right)^3 dy}{V^3}. \tag{12}$$

Experimental computer researches were run according to formulas (8)-(12) and diagrams of the function change $u_x(y,t)$; $V(t)$; $\beta(t)$; $\alpha(t)$; $\tau(y,t)$ depending on Reynolds number were consequently obtained.

The diagrams of instantaneous velocity change according to the effective cross-section depending Reynolds number ($Re = 1, 100, 1000, 1500, 2000$) are given on figures 1-4.

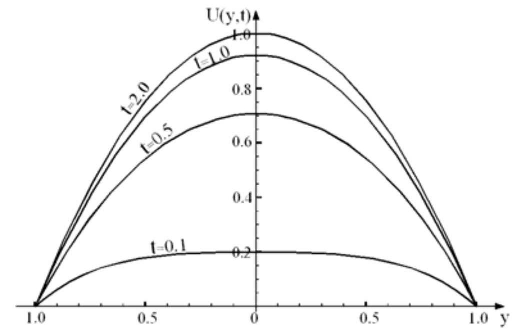

Fig. 1. $Re = 1$, ($t_{ñò} = 1.88$)

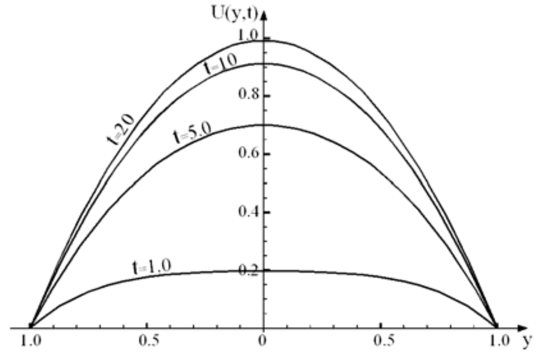

Fig. 2. $Re = 10$, ($t_{ñò} = 18.8$)

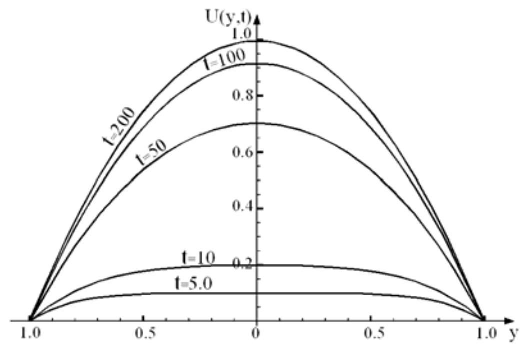

Fig. 3. $Re = 100$, ($t_{ñò} = 188$)

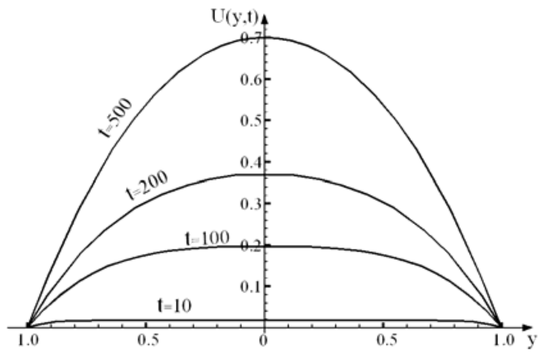

Fig. 4. $Re = 1000$, ($t_{ñò} = 1879.5$)

According to the obtained diagrams, it is obvious that in small Reynolds numbers, transient process quickly strives for steady-state condition and in large Reynolds number values – unsteady condition is maintained for much longer time.

From the diagrams for instantaneous velocity change (fig. 1-4) and shear stresses (fig. 5-8) it is also seen, that in accelerated plane-parallel pressure flow viscous liquid starts its motion by the walls of the fixed canal where boundary layer is formed, and in the centre it moves like solid.

Two zones are formed: the zone of the boundary layer where various layers move with various velocities resulting in the formation of the shear stresses between liquid and main body, and the zone where liquid particles move with equal velocities resulting in the absence of the shear stresses between the liquid layers (fig. 5-8).

Thickness of the boundary layer gradually increases and the diameter of the main body decreases and within the certain time laminar boundary layer fully covers the entire effective cross-section. For each case, time for the process stationarization is indicated when the velocity in flow centre $0{,}99U_{ñò}$.

For these very Reynolds numbers the diagrams of the shear stress change (fig. 5-8) are drafted. From the obtained diagrams it is seen that at the beginning of the nonstationary motion, the shear stresses are formed within the boundary layer zone which gradually

extends and covers the entire effective cross-section. In $t \to t_{ñò}$ the diagrams of the shear stress change strive for linear law.

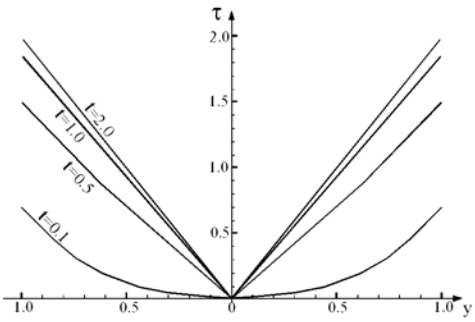

Fig. 5. Re = 1

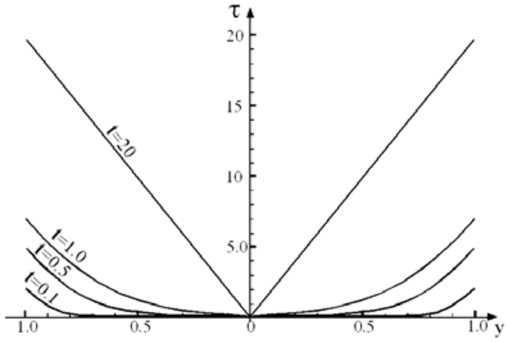

Fig. 6. Re = 10

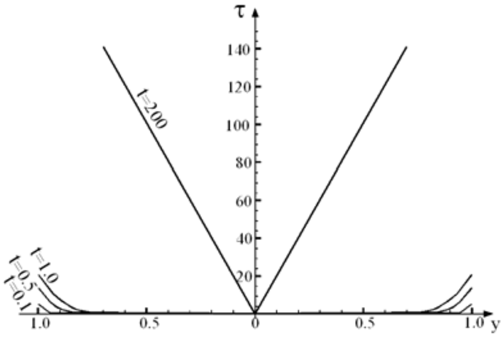

Fig. 7. Re = 100

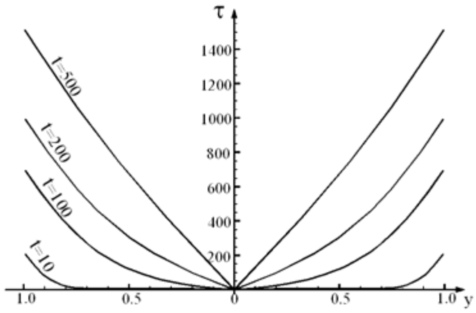

Fig. 8. Re = 1000

The diagrams of the change of momentum coefficient β and kinetic energy α are provided on figures 9 and 10.

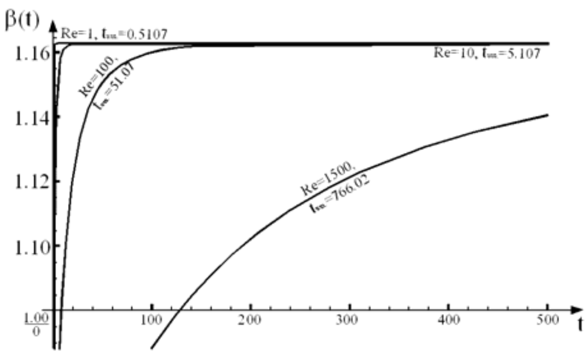

Fig. 9.

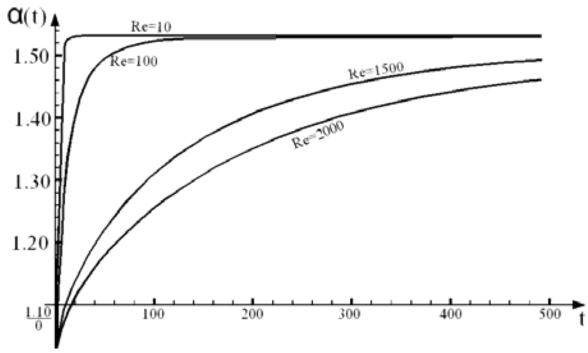

Fig. 10.

In small Reynolds numbers these diagrams quickly strive for the stationary values $\beta_{ñò} \to 1{,}2$, $\alpha_{ñò} = \dfrac{54}{35}$ [1], and in large Reynolds number values, stationarization time increases. The stationarization time is indicated for each diagram.

The diagrams for the change of average velocities of the effective cross-section for various Reynolds numbers are obtained on figure 11. The stationarization time is indicated for each diagram.

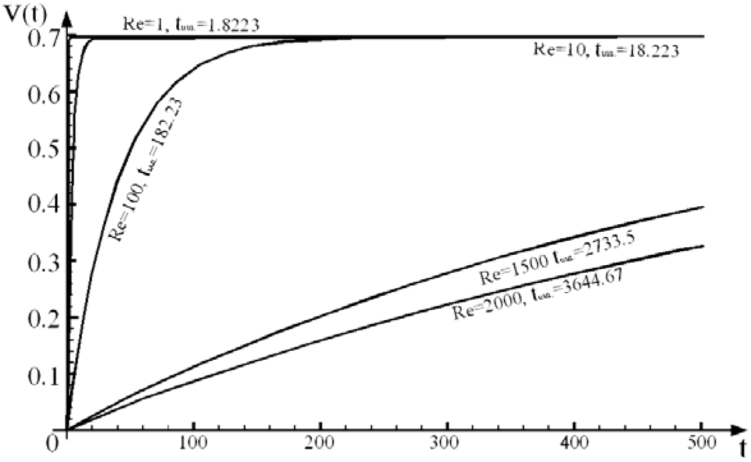

Fig. 11.

Conclusions

1. **The obtained regularities of change of nonstationary hydromechanic parameters of plane-parallel pressure flow under fixed differential pressure allow to identify the nature of the processes occurred and the mechanism of hydraulic losses.**
2. **Criterion for nonstationarity of the plane-parallel pressure flow is Reynolds number.**

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