EQUATIONS INTERCONNECTING EMERGENCY CURRENTS OCCURRED DURING CONCURRENT SYMMETRIC AND ASYMMETRIC SHORT CIRCUIT WITHIN ANY POWER SYSTEM NETWORK

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Interconnecting the equations of emergency currents occurred during concurrent symmetric and asymmetric short circuit within the power system network are obtained. The amount of symmetric short circuits concurrently occurred within the power system network is presented in the networks by switching on such two ideal type voltages (electromotive forces) which are equal to each other in size though are directed reversely.

Equation system is drafted and its solution allows to get direct succession current values at the damage points. For the calculation unification purposes, all the forms of short circuits are considered as the result of one-phase short circuit superposition, and three-phase short circuits – as three one-phase short circuit superposition. Various combinations of concurrently acting asymmetric short circuits are calculated.

Key words: emergency currents, short circuit, intrinsic resistance, superpositions of currents, emergency network.

The amount of symmetric short circuits concurrently occurred within the power system network is presented by switching on such two ideal type voltages (electromotive forces) which are equal to each other in size though are directed reversely. Voltage can be of any size as well as equal to normal mode nodal voltages. The voltage sources with coinciding voltage direction maintain reset conditions unchanged. The voltage sources with the reverse voltage direction create emergency current components in branches as a result of the superposition and create emergency currents at the damage points. Interconnections of the emergency currents are described by matrix equation:

$$Y_{uz}U_{uz} = I_{uz} \qquad \text{or} \qquad Y_{uz}^{-1}I_{uz} = U_{uz} , \qquad (1)$$

where $Y_{uz}^{-1} = Z_{uz}$ – matrix of nodal resistances; U_{uz} – vector-matrix of nodal voltages, where the voltages within the short circuit networks equal to the normal mode voltages; I_{uz} – vector-matrix of nodal currents, where significant elements are the unknown currents of the short circuit and the remainder equals to zero.

Based on the fact that symmetric components of asymmetric emergency current circulate within just the circuits of the respective succession, the dependences between the currents and voltages existing within the symmetric systems are also fair within the separate succession circuits. Respectively, the dependences between the currents and voltages within the separate succession circuits under the condition that the special phase currents (in case of one-phase short circuit – damaged) are brought to phase A through the shift operator can be described with nodal equations like the equations (1).

So, for the currents and voltages in all succession circuits, like (1), in case of one-phase short circuit, the following matrix equations can be made:

$$\begin{bmatrix} Z'_{11} & Z'_{1i} & Z'_{1j} & Z'_{1k} & Z'_{1n} \\ Z'_{11} & Z'_{1i} & Z'_{1j} & Z'_{1k} & Z'_{1n} \\ Z'_{j1} & Z'_{j1} & Z'_{j1} & Z'_{jk} & Z'_{jn} \\ Z'_{k1} & Z'_{ki} & Z'_{kj} & Z'_{kk} & Z'_{kn} \\ Z'_{n1} & Z'_{ni} & Z'_{nj} & Z'_{nk} & Z'_{nn} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ I'_{iA} \\ aI'_{jB} \\ a^{2}I'_{kC} \\ 0 \end{bmatrix} = \begin{bmatrix} U'_{1} \\ u_{iA} - U'_{iA} \\ aU_{jB} - aU'_{jB} \\ a^{2}U_{kC} - a^{2}U'_{kC} \\ U'_{n} \end{bmatrix};$$

$$\begin{bmatrix} Z''_{11} & Z''_{1i} & Z''_{1j} & Z''_{nk} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nk} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nk} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nk} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nk} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nk} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nk} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nk} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nn} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nn} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nn} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nn} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nn} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nn} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nn} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{nj} & Z''_{nn} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{nn} \\ Z''_{n1} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{ni} \\ Z''_{n1} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{ni} \\ Z''_{n1} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{ni} \\ Z''_{n1} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{ni} \\ Z''_{n1} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{ni} \\ Z''_{n1} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{ni} & Z''_{ni} \\ Z'''_{ni} & Z'''_{ni} & Z'''_{ni} & Z'''_{ni} & Z'''_{ni} & Z'''_{ni} \\ Z''''_{ni} & Z'''_{ni} & Z''''_{ni} & Z''''_{ni} & Z''''_{ni} & Z''''$$

In this case the phase A is damages in node i and the phase B - in node j and phase C - in node K. Matrix elements in the equations (2) are the matrix elements of intrinsic and mutual resistances within the circuits of direct, reverse and zero successions.

As a result of multiplying the matrixes in the equation (2), will obtain the equation interconnecting emergency currents and voltages within the circuits of three successions:

$$\begin{cases} Z'_{ii}I'_{iA} + Z'_{ij}aI'_{jB} + Z'_{ik}a^{2}I'_{kC} = U_{iA} - U'_{iA} \\ Z'_{ji}I'_{iA} + Z'_{jj}aI'_{jB} + Z'_{jk}a^{2}I'_{kC} = aU_{jB} - aU'_{jB} \\ Z'_{ki}I'_{iA} + Z'_{kj}aI'_{jB} + Z'_{kk}a^{2}I'_{kC} = a^{2}U_{kC} - a^{2}U'_{kC}. \end{cases}$$

From the mentioned equations the second will be multiplied by a^2 , and the third – by a and will obtain:

$$\begin{cases} Z'_{ii}I'_{iA} + Z'_{ij}aI'_{jB} + Z'_{ik}a^{2}I'_{kC} = U_{iA} - U'_{iA} \\ Z'_{ji}I'_{iA}a^{2} + Z'_{jj}I'_{jB} + Z'_{jk}I'_{kC}a = U_{jB} - U'_{jB} \\ Z'_{ki}I'_{iA}a + Z'_{kj}a^{2}I'_{jB} + Z'_{kk}I'_{kC} = U_{kC} - U'_{kC}. \end{cases}$$
(3)

Analogously, we write the equation for the currents and voltages of the reverse and zero successions as a result of multiplying the relative equations (2):

(2)

$$\begin{cases} Z_{ii}''I_{iA}'' + Z_{jj}''a^2 I_{jB}'' + Z_{ik}''a I_{kC}'' = -U_{iA}''\\ Z_{ji}''a I_{iA}'' + Z_{jj}''I_{jB}'' + Z_{jk}''a^2 I_{kC}'' = -U_{jB}''\\ Z_{ki}''a^2 I_{iA}'' + Z_{kj}''a I_{jB}'' + Z_{kk}''I_{kC}'' = -U_{kC}'' \end{cases} \begin{cases} Z_{ii}^0 I_{iA}^0 + Z_{ij}^0 I_{jB}^0 + Z_{ik}^0 I_{kc}^0 = -U_{iA}^0\\ Z_{ji}^0 I_{iA}^0 + Z_{jj}^0 I_{jB}^0 + Z_{jk}^0 I_{kc}^0 = -U_{jB}^0\\ Z_{ki}^0 I_{iA}^0 + Z_{kj}^0 I_{jB}^0 + Z_{kk}^0 I_{kc}^0 = -U_{kC}^0 \end{cases}$$
(4)

If we summarize the equations for nodes i, j, k made by the circuits of the direct, reverse and zero successions (3), (4), taking into the account the boundary conditions of the one-phase short circuit (5), will get the equation (6):

$$I' = I'' = I^0$$
 and $U' + U'' + U^0 = 0.$ (5)

We got the equation system the decision of which provides the value of the currents of the direct (reverse or zero) succession at the damage points, i.e. in the nodes i, j, k. The arranged values of given currents are the phase values of the emergency currents:

$$\begin{cases} \left(Z'_{ii} + Z''_{ii} + Z^{0}_{ii}\right)I'_{iA} + \left(aZ'_{ij} + a^{2}Z''_{ij} + Z^{0}_{ij}\right)I'_{jB} + \left(a^{2}Z'_{ik} + aZ''_{ik} + Z^{0}_{ik}\right)I'_{kc} = U_{iA} \\ \left(a^{2}Z'_{ji} + aZ''_{ji} + Z^{0}_{ji}\right)I'_{iA} + \left(Z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right)I'_{jB} + \left(aZ'_{jk} + a^{2}Z''_{jk} + Z^{0}_{jk}\right)I'_{kc} = U_{jB} = a^{2}U_{jA} \\ \left(aZ'_{ki} + a^{2}Z''_{ki} + Z^{0}_{ki}\right)I'_{iA} + \left(a^{2}Z'_{kj} + aZ''_{kj} + Z^{0}_{kj}\right)I'_{jB} + \left(aZ'_{kk} + a^{2}Z''_{kk} + Z^{0}_{kk}\right)I'_{kc} = U_{kC} = aU_{kA} \end{cases}$$
(6)

For the circuits of the direct, reverse and zero successions the matrixes of the intrinsic and mutual resistances of the nodes Z', Z'', Z^0 are estimated in advance. Then, the intrinsic and mutual resistances corresponding to the short circuit nodes are selected from these matrixes. If the equation is written for the node i, then the intrinsic resistance of this node and the mutual resistance of the nodes i and j of similar damaged phases will be the sum of the respective intrinsic and mutual resistances within the circuits of all successions:

$$Z'_{ii} + Z''_{ii} + Z^0_{ii} = Z_{iisov} \qquad \text{ and } \qquad Z'_{ij} + Z''_{ij} + Z^0_{ij} = Z_{ijsov},$$

When the damages occur in the opposite phases, then for the calculation of the similar resistances z_{ij} the calculation is made either under the formula:

$$\mathbf{a}^{2}\mathbf{Z}_{ij}' + \mathbf{a}\mathbf{Z}_{ij}'' + \mathbf{Z}_{ij}^{0} = \mathbf{Z}_{ijop}$$

if the damaged phase in the node j leads premature phase in the node i, either under the formula

$$aZ'_{ij} + a^2 Z''_{ij} + Z^0_{ij} = Z_{ijot}$$
.

if the damaged phase in the node j lags behind the damaged phase in the node i. The intrinsic resistances in this case are determined as they were done above.

Like the equation (6), it is possible to make the equations for any amount of concurrent short circuits on the ground in various phases.

For calculation unification purposes, all types of the short circuits should be considered as the results of superposition of one-phase short circuit [3]. For instance, three-phase short circuit is considered as the superposition of three one-phase short circuits. If node merging condition is superpositioned on the equation (6) (in the circuits of all successions)

$$Z_{ii} = Z_{jj} = Z_{kk} = Z_{ij} = Z_{ik} = Z_{kj}$$
(7)

then the solution will give the value of the one third of the phase currents in the node i, if i is the node of the three-phase short circuit. Same happens with two-phase short circuit on

the ground, it is considered as the superposition of two one-phase short circuit. The relative equations considering the condition (7) will have the following form:

$$\begin{array}{l} (\mathbf{Z}'_{ii} + \mathbf{Z}''_{ii} + \mathbf{Z}^{0}_{ii})\mathbf{I}'_{iB} + (\mathbf{a}\mathbf{Z}'_{ii} + \mathbf{a}^{2}\mathbf{Z}''_{ii} + \mathbf{Z}^{0}_{ii})\mathbf{I}'_{iC} = \mathbf{a}^{2}\mathbf{U}_{iA} \\ (\mathbf{a}^{2}\mathbf{Z}'_{ii} + \mathbf{a}\mathbf{Z}''_{ii} + \mathbf{Z}^{0}_{ii})\mathbf{I}'_{iB} + (\mathbf{Z}'_{ii} + \mathbf{Z}''_{ii} + \mathbf{Z}^{0}_{ii})\mathbf{I}'_{iC} = \mathbf{a}\mathbf{U}_{iA} \end{array} \}$$

$$(8)$$

So, all types of asymmetric short circuits on the ground are considered as the superpositions of one-phase short circuits and the relevant unified equations are made. For the insertion of the interphase short circuits in these equations, it is necessary to take into the account additional conditions that are the consequences of such circumstances when one phase, for instance the phase B, is switched to another phase C, i.e. $I_B = -I_C$. Fulfillment of this conditions corresponds to the rotation of the currents of two-phase short circuits at 120⁰ so that the currents of the direct and reverse successions of the phase C in the node j coincide with the direction of the currents of the phase B in the node i (figure 2). For comparing the two-phase short circuit currents on the ground, relevant diagrams of the currents of the direct and reverse successions (figure 1) are drafted. Vector diagrams of the currents of zero succession are not considered as they represent one-phase system.



Fig. 1. Vector diagram within the two-phase short circuit on the ground presented as two one-phase short circuit in the nodes i and j



Fig. 2. Vector diagram within the interphase short circuit presented as two one-phase short circuit in the nodes i and j

The equations of the node i (special phase B) for the circuit of the direct, reverse and zero successions will have the following form:

$$Z'_{ii}I'_{iB} + aZ'_{ij}I'_{jC} = U_{iB} - U'_{iB}$$

$$Z''_{ii}I''_{iB} + aZ''_{ij}I''_{jC} = -U''_{iB}$$

$$Z^{0}_{ii}I^{0}_{iB} + Z^{0}_{ij}I^{0}_{jC} = -U^{0}_{iB}$$
(9)

Summarization of the equations (9) taking into the account the one-phase short circuit condition (4) provides:

$$(Z'_{ii} + Z''_{ii} + Z^0_{ii})I'_{iB} + (aZ'_{ij} + aZ''_{ij} + Z^0_{ij})I'_{jC} = U_{iB} = a^2 U_{iA} .$$
(10)

The equation for the node j according to the circuits of all successions would be:

$$Z'_{jj}aI'_{Cj} + Z'_{ji}I'_{Bi} = aU_{Cj} - aU'_{Cj}$$

$$Z''_{jj}aI''_{Cj} + Z''_{ji}I''_{Bi} = -aU''_{Cj}$$

$$Z^{0}_{jj}I^{0}_{Cj} + Z^{0}_{ji}I^{0}_{Bi} = -U^{0}_{Cj}$$
(11)

Summarization of the equations (11), taking into the consideration (5), after multiplying the first and the second equations of the system (11) by a^2 defines the equation for the node j

$$(\mathbf{Z}'_{jj} + \mathbf{Z}''_{jj} + \mathbf{Z}^{0}_{jj})\mathbf{I}'_{jC} + (\mathbf{a}^{2}\mathbf{Z}'_{ji} + \mathbf{a}^{2}\mathbf{Z}''_{ji} + \mathbf{Z}^{0}_{ji})\mathbf{I}'_{iB} = \mathbf{U}_{jC} = \mathbf{a}\mathbf{U}_{jA}$$

Consequently we will get the equation system describing two-phase short circuit:

$$\left\{ \left\{ Z'_{ii} + Z''_{ii} + Z^{0}_{ii} \right\} I'_{iB} + \left\{ a Z'_{ii} + a Z''_{ii} + Z^{0}_{ii} \right\} I'_{iC} = a^{2} U_{iA} \\ \left\{ a^{2} Z'_{ii} + a^{2} Z''_{ii} + Z^{0}_{ii} \right\} I'_{iB} + \left\{ Z'_{ii} + Z''_{ii} + Z^{0}_{ii} \right\} I'_{iC} = a U_{iA} \right\}$$

$$(12)$$

Solution of (12) gives the one third of the value of phase currents in the short circuit between the phases B and C.

For the identification of the interconnection of the phase currents within the interphase short circuit with the currents in one-phase short circuit in the nodes with damages in various phases, let's firstly review one-phase short circuit in the node i in two-phase short circuit between the phases B and C in the node j.

Let's produce modeling of the currents at the short circuit points with ideal current sources in the circuits of the direct, reverse and zero successions (fig. 3).



Fig. 3.

The equations of the dependence between the currents in the circuits of the direct, reverse and zero successions for the node i where phase A is closed, will have the following form:

N.Turkia,...

$$\begin{cases} Z'_{ii}I'_{iA} + Z'_{ij}aI'_{jB} + Z'_{ij}a^{2}I'_{jC} = U_{iA} - U'_{iA} \\ Z''_{ii}I''_{iA} + Z''_{ij}a^{2}I''_{jB} + Z''_{ij}I''_{jC} = -U''_{iA} \\ Z^{0}_{ii}I^{0}_{iA} + Z^{0}_{ij}I^{0}_{jB} + Z^{0}_{ij}I^{0}_{jC} = -U^{0}_{iA} \end{cases}$$
(13)

Summation of the equations (13) considering the boundary conditions of the one-phase short circuit will give:

$$\left(Z'_{ii} + Z''_{ii} + Z^{0}_{ii} \right) \mathbf{I}'_{iA} + \left(a Z'_{ij} + a^2 Z''_{ij} + Z^{0}_{ij} \right) \mathbf{I}'_{jB} + \left(a^2 Z'_{ij} + Z''_{ij} + Z^{0}_{ij} \right) \mathbf{I}'_{jC} = U_{iA}$$
(14)

Analogously, for the node j, where the phase B is closed, will obtain:

$$\begin{cases} Z'_{ji}a^{2}I'_{iA} + Z'_{jj}I'_{jB} + Z'_{jj}aI'_{jC} = U_{jB} - U'_{jB} \\ Z''_{ji}aI''_{iA} + Z''_{jj}I''_{jB} + Z''_{jj}aI''_{jC} = -U''_{jB} \\ Z^{0}_{ji}I^{0}_{iA} + Z^{0}_{jj}I^{0}_{jB} + Z^{0}_{jj}I^{0}_{jC} = -U^{0}_{jC} \end{cases}$$
(15)

$$\left(a^{2}Z'_{ji} + aZ''_{ji} + Z^{0}_{ji}\right)I'_{iA} + \left(Z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right)I'_{jB} + \left(aZ'_{jj} + aZ''_{jj} + Z^{0}_{jj}\right)I'_{jC} = U_{jB}$$
(16)

For the node j, where the phase C is closed, will obtain:

$$\begin{cases} Z'_{ji} a I'_{iA} + Z'_{jj} a^{2} I'_{jB} + Z'_{jj} I'_{jC} = U_{jC} - U'_{jC} \\ Z''_{ji} a^{2} I''_{iA} + Z''_{jj} a I''_{jB} + Z''_{jj} a^{2} I''_{jC} = -a^{2} U''_{jC} \\ Z^{0}_{ji} I^{0}_{iA} + Z^{0}_{jj} I^{0}_{jB} + Z^{0}_{jj} I^{0}_{jC} = -U^{0}_{jC} \end{cases}$$

$$(17)$$

$$(a Z'_{ji} + Z''_{ji} + Z^{0}_{ji}) I'_{iA} + (a^{2} Z'_{jj} + a^{2} Z''_{jj} + Z^{0}_{jj}) I'_{jB} + (Z'_{jj} + Z''_{jj} + Z^{0}_{jj}) I'_{jC} = U_{jC}.$$

The equation system determining the sought unknowns will have the following form:

$$\begin{cases} \left(Z'_{ii} + Z''_{ii} + Z^{0}_{ii}\right)I'_{iA} + \left(aZ'_{ij} + a^{2}Z''_{ij} + Z^{0}_{ij}\right)I'_{jB} + \left(a^{2}Z'_{ij} + Z''_{ij} + Z^{0}_{ij}\right)I'_{jC} = U_{iA} \\ \left(a^{2}Z'_{ji} + aZ''_{ji} + Z^{0}_{ji}\right)I'_{iA} + \left(Z'_{jj} + Z''_{jj} + Z^{0}_{ji}\right)I'_{jB} + \left(aZ'_{jj} + aZ''_{jj} + Z^{0}_{jj}\right)I'_{jC} = a^{2}U_{jA} \\ \left(aZ'_{ji} + Z''_{ji} + Z^{0}_{ji}\right)I'_{iA} + \left(a^{2}Z'_{jj} + a^{2}Z''_{jj} + Z^{0}_{jj}\right)I'_{jB} + \left(Z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right)I'_{jC} = aU_{jA} \\ I^{F}_{iA} = 3I'_{iA}, \qquad I^{F}_{jB} = 3I'_{jB}, \qquad I^{F}_{jC} = 3I'_{jC} \end{cases}$$

$$(18)$$

The equations (19) determining the sought values I'_{iB} , I'_{jB} , I'_{jC} , when the phase B is closed in the node i and the phases B and C in the node j are closed between each other (fig. 4) are obtained in a similar way.

$$\begin{cases} \left(Z'_{ii} + Z''_{ii} + Z^{0}_{ii}\right)I'_{iB} + \left(Z'_{ij} + Z''_{ij} + Z^{0}_{ij}\right)I'_{jB} + \left(aZ'_{ij} + aZ''_{ij} + Z^{0}_{ij}\right)I'_{jC} = a^{2}U_{iA} \\ \left(Z'_{ji} + Z''_{ji} + Z^{0}_{ji}\right)I'_{iB} + \left(Z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right)I'_{jB} + \left(aZ'_{jj} + aZ''_{jj} + Z^{0}_{jj}\right)I'_{jC} = a^{2}U_{iA} \\ \left(a^{2}Z'_{ji} + a^{2}Z''_{ji} + Z^{0}_{ji}\right)I'_{iB} + \left(a^{2}Z'_{jj} + a^{2}Z''_{jj} + Z^{0}_{jj}\right)I'_{jB} + \left(Z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right)I'_{jC} = aU_{jA} \end{cases}$$
(19)

$$Z_{ii} = Z_{mm} = Z_{nn} = Z_{im} = Z_{in} = Z_{mn}$$
(20)

,



Fig. 4.

The equations for the case when the phase C is closed in the node i and the phases B and C are closed between each other in the node j (fig. 5) are obtained in a similar way

$$\begin{cases} \left(Z'_{ii} + Z''_{ii} + Z^{0}_{ii}\right)I'_{iC} + \left(a^{2}Z'_{ij} + aZ''_{ij} + Z^{0}_{ij}\right)I'_{jB} + \left(Z'_{ij} + a^{2}Z''_{ij} + Z^{0}_{ij}\right)I'_{jC} = aU_{iA} \\ \left(aZ'_{ji} + a^{2}Z''_{ji} + Z^{0}_{ji}\right)I'_{iC} + \left(Z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right)I'_{jB} + \left(aZ'_{jj} + aZ''_{jj} + Z^{0}_{jj}\right)I'_{jC} = a^{2}U_{jA} \\ \left(Z'_{ji} + aZ''_{ji} + Z^{0}_{ji}\right)I'_{iC} + \left(a^{2}Z'_{jj} + a^{2}Z''_{jj} + Z^{0}_{jj}\right)I'_{jB} + \left(Z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right)I'_{jC} = aU_{jA} \end{cases}$$

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Various combinations of the concurrent symmetric short circuits are calculated, however the case of the interphase short circuit in the node j in case of three-phase short circuit in the node i was chosen as the most illustrative example showing the calculation consistency based on the suggested method. The condition of merging the nodes i, m, n (20) was superpositioned on the equation for three one-phase short circuit in the nodes i, m, n (for two-phase short circuit in the node j, the condition of the node merging was considered prior to making the equations) (21). Consequently, the equations (22) were obtained:



Fig. 5

So, the equations are made for five one-phase short circuits – two one-phase short circuits in the node j and three one-phase short circuits in the node i. The intrinsic resistances for the one-phase short circuit in the node i in the circuits of all successions equal to Z'_{ii} , Z''_{ii} , Z^{0}_{ii} , in the node j j $-Z'_{jj}$, Z''_{jj} , Z^{0}_{jj} , mutual resistances between the nodes i and j would be Z'_{ij} , Z''_{ij} , Z''_{ij} .

N.Turkia,...

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$$\begin{bmatrix} \left(Z'_{ii} + Z''_{ii} + Z^{0}_{ii}\right) & \left(aZ'_{im} + a^{2}Z''_{im} + Z^{0}_{im}\right) & \left(a^{2}Z'_{in} + aZ''_{in} + Z^{0}_{in}\right) & \left(aZ'_{ij} + a^{2}Z''_{ij} + a^{2}Z''_{ij} + Z^{0}_{ij}\right) & \left(a^{2}Z'_{ij} + Z^{0}_{ij} + Z^{0}_{ij}\right) \\ \left(a^{2}Z'_{mi} + aZ''_{mi} + Z^{0}_{mi}\right) & \left(Z'_{mm} + Z''_{mm} + Z^{0}_{mm}\right) & \left(aZ'_{mn} + a^{2}Z''_{mn} + Z^{0}_{mn}\right) & \left(Z'_{mj} + Z''_{mj} + Z^{0}_{mj}\right) & \left(aZ'_{mj} + aZ''_{mj} + Z^{0}_{mj}\right) \\ \left(aZ'_{ni} + a^{2}Z''_{ni} + Z^{0}_{ni}\right) & \left(a^{2}Z'_{nm} + aZ''_{mm} + Z^{0}_{nm}\right) & \left(Z'_{nn} + Z''_{nn} + Z^{0}_{nn}\right) & \left(a^{2}Z'_{nj} + aZ''_{nj} + Z^{0}_{nj}\right) & \left(Z'_{nj} + a^{2}Z''_{nj} + Z^{0}_{nj}\right) \\ \left(aZ'_{ji} + aZ''_{ji} + Z^{0}_{ji}\right) & \left(Z'_{jm} + Z''_{jm} + Z^{0}_{jm}\right) & \left(aZ'_{jn} + a^{2}Z''_{jn} + Z^{0}_{jn}\right) & \left(Z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right) & \left(Z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right) \\ \left(aZ'_{ji} + Z''_{ji} + Z^{0}_{ji}\right) & \left(a^{2}Z'_{jm} + a^{2}Z''_{jm} + Z^{0}_{jm}\right) & \left(Z'_{jn} + aZ''_{jn} + Z^{0}_{jn}\right) & \left(a^{2}Z'_{jj} + aZ''_{jj} + Z^{0}_{jj}\right) & \left(Z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right) \\ \left(aZ'_{ji} + Z''_{ji} + Z^{0}_{ji}\right) & \left(a^{2}Z'_{jm} + a^{2}Z''_{jm} + Z^{0}_{jm}\right) & \left(Z'_{jn} + aZ''_{jn} + Z^{0}_{jn}\right) & \left(a^{2}Z'_{jj} + aZ''_{jj} + Z^{0}_{jj}\right) & \left(Z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right) \\ \left(aZ'_{ji} + Z''_{ji} + Z^{0}_{ji}\right) & \left(a^{2}Z'_{jm} + a^{2}Z''_{jm} + Z^{0}_{jm}\right) & \left(Z'_{jn} + aZ''_{jn} + Z^{0}_{jn}\right) & \left(a^{2}Z'_{jj} + aZ''_{jj} + Z^{0}_{jj}\right) & \left(Z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right) \\ \left(z'_{jj} + Z''_{jj} + Z^{0}_{ji}\right) & \left(z'_{jj} + z''_{jj} + Z^{0}_{jm}\right) & \left(Z'_{jn} + z''_{jj} + Z^{0}_{jj}\right) & \left(Z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right) & \left(Z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right) \\ \left(z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right) & \left(z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right) & \left(z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right) & \left(z'_{jj} + Z''_{jj} + Z^{0}_{jj}\right) \\ \left(z'_{jj} + Z''_{jj} + Z''_{jj}\right) & \left(z'_{jj} + Z''_{jj} + Z''_{jj}\right) \\ \left(z'_{jj} + Z''_{jj} + Z''_{jj}\right) & \left(z'_{jj} + Z$$

As a result of considering the condition of the node merging for the node with three-phase short circuits will obtain:

$$\begin{bmatrix} \left(Z'_{ii} + Z''_{ii} + Z^{0}_{ii}\right) & \left(aZ'_{ii} + a^{2}Z''_{ii} + Z^{0}_{ii}\right) & \left(a^{2}Z'_{ii} + aZ''_{ii} + Z^{0}_{ii}\right) & \left(aZ'_{ij} + aZ''_{ij} + Z^{0}_{ij}\right) & \left(a^{2}Z'_{ij} + Z''_{ij} + Z^{0}_{ij}\right) & \left(a^{2}Z'_{ij} + Z''_{ij} + Z^{0}_{ii}\right) & \left(a^{2}Z'_{ij} + aZ''_{ij} + Z^{0}_{ij}\right) & \left(aZ'_{ij} + Z''_{ij} + Z^{0}_{ij}\right) & \left(aZ'_{ij} + aZ''_{ij} + Z^{0}_{ij}\right) & \left(aZ'_{ij} + Z''_{ij} + Z^{0}_{ij}\right) & \left(aZ'_{ij} + aZ''_{ij} + Z^{0}_{ij}\right) & \left(Z'_{ij} + Z''_{ij} + Z''_{ij} + Z^{0}_{ij}\right) & \left(Z''_{ij} + Z''_{ij} + Z''_{ij} + Z'$$



This example is convenient, because for justifying the validity of the obtained results, they can be compared to the results of the calculations run based on ordinary method for one damage point, i.e. the calculations of two-phase short circuit in the node 3 in an immediate closure of the node 2 on the ground. In this case, the influence of the short circuit is considered by recalculation of the intrinsic and mutual resistances of the nodes based on the changes caused by the closure of the node 2 on the ground.

The intrinsic and mutual resistances of the nodes 2 and 3 for the circuit of the direct, reverse and zero successions calculated in MATLAB are presented in form of the matrix Z1, Z2, Zo.

$$Z1 = \begin{bmatrix} Z'_{22} & Z'_{23} \\ Z'_{32} & Z'_{33} \end{bmatrix} = \begin{bmatrix} 0 + j0.7286 & 0 + j0.5476 \\ 0 + j0.5476 & 0 + j0.7286 \end{bmatrix}$$
$$Z2 = \begin{bmatrix} Z''_{22} & Z''_{23} \\ Z''_{32} & Z''_{33} \end{bmatrix} = \begin{bmatrix} 0 + j0.3842 & 0 + j0.2191 \\ 0 + j0.2191 & 0 + j0.3842 \end{bmatrix}$$
$$Z0 = \begin{bmatrix} Z^0_{22} & Z^0_{23} \\ Z^0_{32} & Z^0_{33} \end{bmatrix} = \begin{bmatrix} 0 + j0.7800 & 0 + j0.2100 \\ 0 + j0.2100 & 0 + j0.7800 \end{bmatrix}$$

It is obvious from the equations of the interconnection of the emergency currents, that the factors for coinciding lagging and leading phases with regards to the special phase of the node for which the equation is written, are calculated with the matrix elements Z1, Z2, Z0 and the vector shift operator a = -0.5 + j0.866 according to the below given formulas:

$$\begin{split} Z_{sov} &= Z1 + Z2 + Z0 \\ Z_{sov} &= Z_{1} + Z_{2} + Z0 \\ Z_{sov} &= \begin{bmatrix} 0 + j1.8928 & 0 + j0.9767 \\ 0 + j0.9767 & 0 + j1.8928 \end{bmatrix} \\ \end{split} \\ \begin{aligned} Z_{ot} &= a \cdot Z1 + a^{2} \cdot Z2 + Z0 \\ Z_{ot} &= a^{2} \cdot Z1 + a \cdot Z2 + Z0 \\ Z_{op} &= a^{2} \cdot Z1 + a \cdot Z2 + Z0 \\ Z_{op} &= \begin{bmatrix} 0.2983 + j0.2236 & -0.2845 - j0.1733 \\ -0.2845 - j0.1733 & -0.2983 + j0.2236 \end{bmatrix} \\ ZZ_{ot} &= a \cdot Z1 + a \cdot Z2 + Z0 \\ ZZ_{ot} &= a \cdot Z1 + a \cdot Z2 + Z0 \\ ZZ_{ot} &= a^{2} \cdot Z1 + a^{2} \cdot Z2 + Z0 \\ ZZ_{op} &= a^{2} \cdot Z1 + a^{2} \cdot Z2 + Z0 \\ ZZ_{op} &= \begin{bmatrix} 0.9637 + j0.2236 & -0.6640 - j0.1733 \\ -0.6640 - j0.1733 & -0.9637 + j0.2236 \end{bmatrix} \\ ZZ_{op} &= a^{2} \cdot Z1 + a^{2} \cdot Z2 + Z0 \\ ZZ_{op} &= \begin{bmatrix} 0.9637 + j0.2236 & 0.6640 - j0.1733 \\ -0.6640 - j0.1733 & 0.9637 + j0.2236 \end{bmatrix} \\ A_{op} &= a^{2} \cdot Z1 + Z2 + Z0 \\ A_{op} &= \begin{bmatrix} 0.6310 + j0.7999 & 0.4742 + j0.1553 \\ 0.4742 + j0.1553 & 0.6310 + j0.7999 \end{bmatrix} \\ B_{op} &= Z1 + a^{2} \cdot Z2 + Z0 \\ A_{ot} &= a \cdot Z1 + Z2 + Z0 \\ A_{ot} &= \begin{bmatrix} -0.6310 + j0.7999 & -0.4742 + j0.1553 \\ -0.4742 + j0.1553 & -0.6310 + j0.7999 \end{bmatrix} \\ B_{ot} &= Z1 + a \cdot Z2 + Z0 \\ B_{ot} &= \begin{bmatrix} -0.3327 + j1.3165 & -0.1897 + j0.6481 \\ -0.1897 + j0.6481 & -0.3327 + j1.3165 \end{bmatrix} \\ A_{ot} &= \begin{bmatrix} -0.3327 + j1.3165 & -0.1897 + j0.6481 \\ -0.1897 + j0.6481 & -0.3327 + j1.3165 \end{bmatrix} \\ \end{bmatrix}$$

Choosing the required elements by indexes of the relevant emergency nodes from the listed matrixes, will make the equation system

$$\mathbf{Z}^{*}\mathbf{I}=\mathbf{U},$$
 (23)

where

$$Z = \begin{bmatrix} [0 + j1.8928 - 0.2983 + j0.2236 & 0.2983 + j0.2236 & -0.2845 - j0.1733 & 0.4742 + j0.1553 \\ 0.2983 + j0.2236 & 0 + j1.8928 & -0.2983 + j0.2236 & 0 + j0.9767 & -0.6640 - j0.1733 \\ -0.2983 + j0.2236 & 0.2983 + j0.2236 & 0 + j1.8928 & 0.2845 - j0.1733 & 0.1897 + j0.6481 \\ 0.2845 - j0.1733 & 0 + j0.9767 & -0.2845 - j0.1733 & 0 + j1.8928 & -0.9637 + j0.2236 \\ -0.4742 + j0.1553 & 0.6640 - j0.1733 & -0.1897 + j0.6481 & 0.9637 + j0.2236 & 0 + j1.8928 \end{bmatrix}$$

$$U = \begin{bmatrix} U_2 \\ a^2 U_2 \\ a U_2 \\ a^2 U_3 \\ a U_3 \end{bmatrix} = \begin{bmatrix} 0.8080 + j0 \\ -0.4040 - j0.6997 \\ -0.4040 + j0.6997 \\ -0.4040 - j0.6997 \\ -0.4040 + j0.6997 \end{bmatrix} - \text{ phase currents of the second and the third nodes of Fig. 6.}$$

Solving the system of the equations (23), will obtain:

$$I = inv(Z) \cdot U$$

$$I = \begin{bmatrix} -0.0573 - j0.3155 \\ -0.1872 + j0.1081 \\ 0.2445 + j0.2074 \\ -0.2011 + j0.0000 \\ 0.2011 + j0.0000 \end{bmatrix}$$

Triple values of 1 matrix elements are the values of the phase currents within the damaged points. For the validity of the suggested method the calculation of two-phase short circuit in the node 3 in the immediate closure of the node 2 on the ground was made. In such case $Z'_{33} = 0 + j0.317$ $Z''_{33} = 0 + j0.2593$ and voltage in the node 3 - $U_3 = 0.2008$. The calculation is made based on the method provided in [1].

$$\begin{split} i_{3B} &= -j1.732 \cdot 0.2008 / (j0.317 + j0.2593) = -0.6035 \\ i_{3C} &= j1.732 \cdot 0.2008 / (j0.317 + j0.2593) = 0.6035 \\ i_{3B} / 3 &= -0.6035 / 3 = -0.2011 \\ i_{3C} / 3 &= 0.2011 \end{split}$$

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