PERIODIC LONGITUDINAL WAVES IN TRAPEZOIDAL CHANNELS

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Alongshore waves are dominant in open river and maritime canals. For water area, from one side limited by arbitrarily sloped shore endlessly running deep into the sea, they were studied by Stocks, but due to significant mathematic difficulties for total depth canals the amount of accurate solutions is limited by just some private cases which are hardly applicable in practical use.

Some results of approximate solution of the problem about propagation of alongshore waves laying over the stationary flow in trapezoid canal are presented. The solution is based on the application of direct Galorkin-Kantorovich method in three-dimension linear equations for wave hydromechanics written in cylindrical coordinate system. The obtained solutions maintain the three-dimension structure of the waves over the shore slope and lead to the results easily applied in the design.

Key words: *alongshore waves, flow, canal depth, slope wetting height.*

Longitudinal waves are the dominating ones in open maritime and navigation river channels. For a area having only one side bounded by an arbitrarily sloping wall, these waves were studied by Stokes [1], but because of great mathematical difficulties for channels of finite depth the quantity of exact solutions is limited only to a few particular cases [1] - [4] which are difficult for practical use.

Below we present some of our results of approximate solution of the problem on propagation of longitudinal waves imposed on a stationary flow in a trapezoidal channel. The solution is based on the application of the Galerkin-Kantorovich direct method [5] to three-dimensional linear equations of wave hydromechanics written in a cylindrical system of coordinates x,r,α, (see Fig. 1)

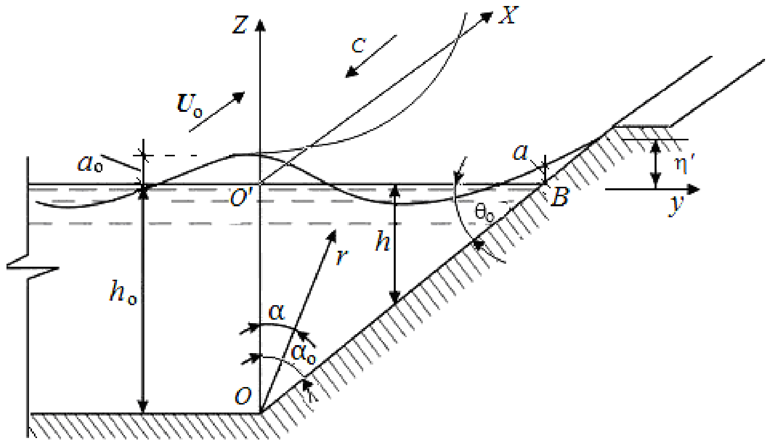

Fig. 1. Design diagram of alongshore waves in trapezoidal channel

where x is a longitudinal coordinate; r is the radius vector taking its origin on the line of intersection of the bank slope with the channel bottom and acting in the sector bounded by the vertical z-axis and the bank slope θ_0 towards the horizon; α is a polar angle that varies from α=0 on the z-axis to $\alpha=\alpha_0$ on the bank slope plane. The following expressions were obtained for the velocity potential (φ) and vertical deviations of the free surface (η) of longitudinal waves:

$$\varphi = U_0 x \pm a_0 \frac{g}{\sigma - kU_0} \frac{\cosh(kr)}{\cosh(kh_0)} \frac{\cos m(\alpha - \alpha_0)}{\cos(m\alpha_0)} \cos(\sigma t \pm kx) \; ; \qquad (1)$$

$$\eta = a_0 \frac{\cosh(kh_0 / \cos\alpha)}{\cosh(kh_0)} \frac{\cos m(\alpha - \alpha_0)}{\cos(m\alpha_0)} \sin(\sigma t \pm kx), \qquad (2)$$

where U_0 is the stationary flow velocity; h_0 and a_0 are respectively the flow depth and the wave amplitude given in the rectangular part of the channel; $\sigma = 2\pi/\tau$ is the wave disturbance frequency; τ is the period of time; $k = 2\pi/\lambda$ is the wave number; λ is the length of a longitudinal wave; the signs "$\pm$" correspond to the propagation of counter-flow waves and waves whose direction coincides with that of a flow; *m* is the so-called transverse wave number on which depends the wave surface configuration crosswise the channel. In particular, if in a channel there propagate relatively short waves for which the number *m* is defined by the asymptotic relation

$$m = \left(\frac{kh_0}{\cos\alpha_0} - \frac{1}{2} \right)^{1/2}, \qquad (3)$$

then the free water surface acquires the mode of standing wave oscillations. These oscillations have longitudinal stationary nodal lines, the number of which over the bank slope is calculated by the integer part of the number *n* defined by the equality

$$n = \frac{m\alpha_0}{\pi} + \frac{1}{2}. \qquad (4)$$

For all values of *n* we use the limit dispersion relation

$$(\sigma - kU_0)^2 = gk\cos\alpha_0 \cdot \tanh(kh_0 / \cos\alpha_0), \qquad (5)$$

Whereas the connection between the wave amplitudes on the bank line *a* and above the bank slope base a_0 is expressed by the relation

$$\frac{a}{a_0} = \left| \frac{\cosh(kh_0 / \cos\alpha_0)}{\cosh kh_0 \cdot \cos m\alpha_0} \right|, \qquad (6)$$

according to which *a* is always larger than a_0 and much exceeds a_0 in the presence of short waves, i.e. for large kh_0. In that case, the equation of free surface near the bank asymptotically leads to the results calculated by the Stokes relation. Also, if the wave steepness on the shore line is

$$\frac{a}{\lambda} \geq \frac{\tan\theta_0}{2\pi e \cos\theta_0}, \qquad (7)$$

where $e = 2.718$ is the Neper number, then above the shoreline waves will wet the bank slope only with destroyed crests. Graphic picture (7) is provided in Fig. 2.

According to Fig.1. Even for very steep coast slope, in particular, for the slope with an angle dipping to the horizon $\theta_0 = 60^o$, the maximum steepness of the alongshore waves at the shoreline does not exceed $\frac{a}{\lambda} = 0.2$. In larger alongshore wave steepness, above the shoreline, will wet the slope of the slope at 60° only with broken ridges.

Using these relations and estimating the static stability of the bank slope of a trapezoidal channel built of loose soil, we can come to a conclusion that by washing-out the

bank slope of the channel short longitudinal waves give it a convex shape, whereas relatively long waves give a concave shape.

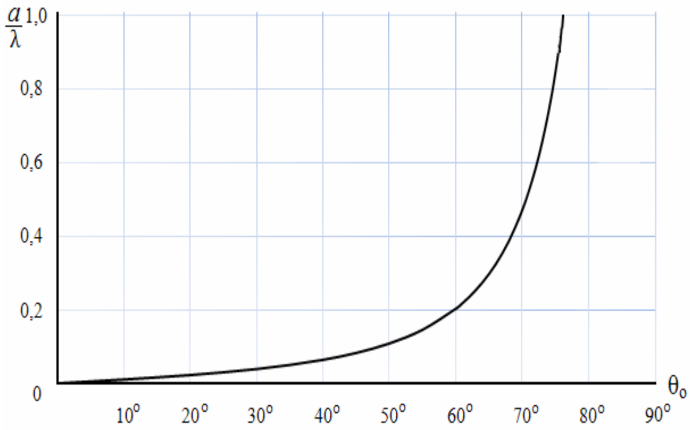

Fig. 2. Limit steepness of the wave crest above the shoreline of trapezoidal channel

R E F E R E N C E S

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