CALCULATION OF STRENGTH OF ARCH DAMS WITH A GENERAL CONTOUR AND OPTIMIZATION OF THEIR VOLUME WITH A COMBINED NUMERICAL METHOD

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The work describes the issues of optimal design of arched dams, when in terms of the geometry of the middle surface, the thicknesses of the design sections of a dam ensuring the equal dam resistance or minimum dam volume are determined. A system of decision equations is presented with five balance equations, three terms of continuity of speed of deformation and one term of complex stressed mode. After the solution of the system, eight internal power factors and optimal thickness are determined unilaterally. The tensions on the contact surface "Dam-base" are determined by the method of finite elements.

Key words: ***dam, mode of deformation, optimization, arch, shell, finite element, finite difference.***

Determination of the geometry and parameters of modern-design arched dams at the initial stage of design is undertaken based on appropriate analogues and recommendations, with further precisions made via numerical experiments, based on the analysis of its deflected mode. The geometrical parameters of an arched dam identified in this manner are often far from their optimal values what is undoubtedly proved by equipotential curves drafted in the dam body, which is being exploited and designed on the condition of strength [2].

The analysis of the deflected mode of dams shows that their different sites are characterized by sharply different reserve coefficients on strength. Accordingly, the mechanical properties of the material in the dam body are not maximally used and the determination of the geometrical properties of the dam needs further research.

Recently, in the construction of dams, particular attention is paid to the acceleration of dam construction and significant reduction of the construction cost, based on optimization of the structure approved by calculations. A general criterion of optimal designing of structures is often its cost, but considering all influential factors is a very complex task.

In practice, they often use a criterion, such as minimum weight (volume) in terms of strength, stability and rigidity of a structure. Sometimes, a decisive importance is attributed not to the minimum weight of the structure, but to the construction technology. Therefore, an issue to create an optimal structure is an urgent and complex problem.

When designing arched dams, aiming at fixing its parameters, it is often necessary to make repeated calculations and analyze the gained results. The desirable results with the statistically definable arched dams may be reached through a single analysis.

The tasks of optimization are mostly solved for such structures, whose calculation scheme is presented in a one-dimension system, i.e. when their decision equations are given in ordinary differential equations. Recently, the tasks of partial optimization (e.g. method of given pressures) are solved for some structures, whose behavior under the stress is described by differential equations with particular derivative. It ensures only the partial improvement of the structure.

There are only some works about the improvement of arched dams known to present. These works are mostly based on numerical experiments and are realized by a complex and labor-consuming iteration cycle.

In order to characterize the deflected mode of an arched dam, as that of a shell, particularly at the initial stage of design (the I and II phases), when a dam type and principal parameters are selected for feasibility study purposes, the theory of shells is more purposeful to use. The use of this method of calculation is complicated by the complexity of the solution to the decision equations due to its complex boundary terms and geometry. The calculation of a "dam-base", as that of a single system is also complicated, particularly when the base ground is non-homogenous, has cracks and sometimes cavities. Calculating a system "dam-base" by a finite element method is not a bit difficult even in case of interrupted parameters.

The deflected mode of a dam body in the work is evaluated by using a shell theory, with the boundary values along the contour of its boundary area fixed for the system "dam-base".

Identifying the unknown thicknesses of the dam is planned from the condition of the complex deflected mode, which ensures the term of fluidity on stretching and compression of the material at every point by considering different resistance values. Such arched dams may be atributed to the category of the same strength what identifies the minimum dam volume (weight) based on the maximum use of the mechanical properties of the material on the one hand and ensures the correctness of the mathematical problem on the other hand.

Instant transition of an arched dam into the plastic state may be ensured even in terms of the given load and thicknesses, based on the rational shape of its median surface, but this time a structure may turn out to be technologically unecceptable.

Identifying the deflected mode of the dam body and optimal thicknesses in the design sections is possible by using the general theory of rigid shells, through a system of variable-coefficient decision equations, which is presented in the work as five balance equations [2], continuity equation of the median surface deformation speed of three shells [6] and condition of strength of complex tensioned state for brittle-ductile materials [7,8]. A system of decision equations within the cylindrical coordinate system (Fig. 1) after appropriate simple transformations will be as follows:

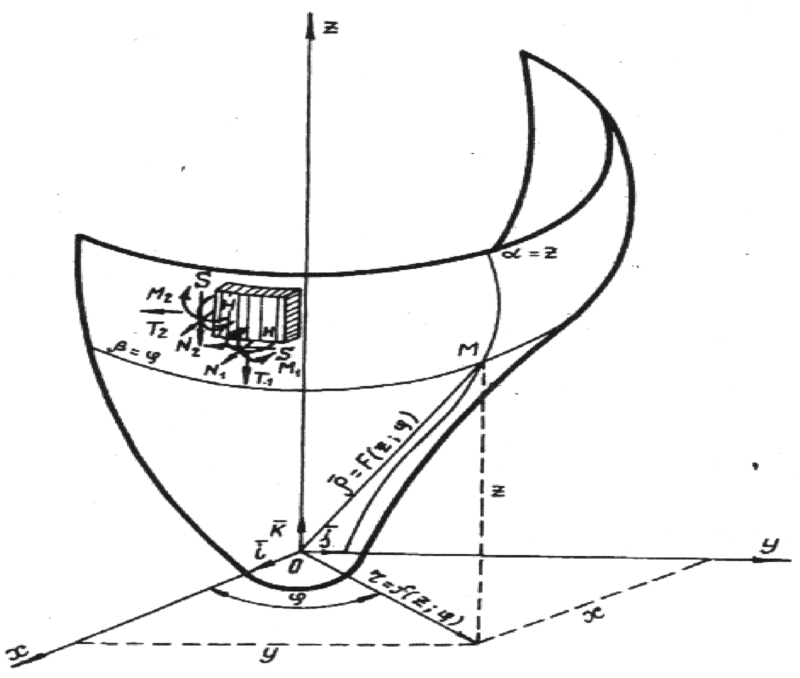

Fig. 1. Calculation plan of an arched dam

$$\frac{\partial A_2}{\partial z}T_1 + A_2\frac{\partial T_1}{\partial z} + 2\frac{\partial A_1}{\partial \varphi}S + A_1\frac{\partial S}{\partial \varphi} - \frac{\partial A_2}{\partial z}T_2 + \frac{A_1A_2}{R_1}N_1 + A_1A_2q_1 = 0;$$

$$2\frac{\partial A_2}{\partial z}S + A_2\frac{\partial S}{\partial z} + \frac{\partial A_1}{\partial \varphi}T_2 + A_1\frac{\partial T_2}{\partial \varphi} - \frac{\partial A_1}{\partial \varphi}T_1 + \frac{A_1A_2}{R_2}N_2 + A_1A_2q_2 = 0;$$

$$\frac{\partial A_2}{\partial z}N_1 + A_2\frac{\partial N_1}{\partial z} + \frac{\partial A_1}{\partial \varphi}N_2 + A_1\frac{\partial N_2}{\partial \varphi} - \frac{A_1A_2}{R_1}T_1 - \frac{A_1A_2}{R_2}T_2 + A_1A_2q_3 = 0;$$

$$\frac{\partial A_2}{\partial z}M_1 + A_2\frac{\partial M_1}{\partial z} + 2\frac{\partial A_1}{\partial \varphi}H + A_1\frac{\partial H}{\partial \varphi} - \frac{\partial A_2}{\partial z}M_2 - A_1A_2N_1 = 0;$$

$$2\frac{\partial A_2}{\partial z}H + A_2\frac{\partial H}{\partial z} + \frac{\partial A_1}{\partial \varphi}M_2 + A_1\frac{\partial M_2}{\partial \varphi} - \frac{\partial A_1}{\partial \varphi}M_1 - A_1A_2N_2 = 0.$$

$$A_2\frac{\partial M_2}{\partial z} + \left(\frac{\partial A_2}{\partial z} - \frac{A_2}{h^3}\frac{\partial h^3}{\partial z}\right)M_2 - \frac{\partial A_2}{\partial z}M_1 - \frac{A_1}{2}\frac{\partial H}{\partial \varphi} +$$

$$+\left(\frac{A_1}{2h^3}\frac{\partial h^3}{\partial \varphi} - \frac{\partial A_1}{\partial \varphi}\right)H + \left(\frac{h^2}{6R_2}\frac{\partial A_1}{\partial \varphi} - \frac{A_1h}{6R_1}\frac{\partial h}{\partial \varphi} + \frac{h^2}{6R_1}\frac{\partial A_1}{\partial \varphi}\right)S +$$

$$+\left(\frac{A_2h}{12R_1}\frac{\partial h}{\partial z} - \frac{h^2}{12R_1}\frac{\partial A_2}{\partial z}\right)T_2 + \frac{h^2}{12R_1}\frac{\partial A_2}{\partial z}T_1 +$$

$$+\frac{A_1h^2}{6R_1}\frac{\partial S}{\partial \varphi} - \frac{A_2h^2}{12R_1}\frac{\partial T_2}{\partial z} = 0;$$

$$A_1\frac{\partial M_1}{\partial \varphi} + \left(\frac{\partial A_1}{\partial \varphi} - \frac{A_1}{h^3}\frac{\partial h^3}{\partial \varphi}\right)M_1 - \frac{\partial A_1}{\partial \varphi}M_2 - \frac{A_2}{2}\frac{\partial H}{\partial z} +$$

$$+\left(\frac{A_2}{2h^3}\frac{\partial h^3}{\partial z} - \frac{\partial A_2}{\partial z}\right)H + \left(\frac{h^2}{6R_1}\frac{\partial A_2}{\partial z} - \frac{A_2h}{6R_2}\frac{\partial h}{\partial z} + \frac{h^2}{6R_2}\frac{\partial A_2}{\partial z}\right)S + \qquad \textbf{(1)}$$

$$+\left(\frac{A_1h}{12R_2}\frac{\partial h}{\partial \varphi} - \frac{h^2}{12R_2}\frac{\partial A_1}{\partial \varphi}\right)T_1 + \frac{h^2}{12R_2}\frac{\partial A_1}{\partial \varphi}T_2 +$$

$$+\frac{A_2h^2}{6R_2}\frac{\partial S}{\partial z} - \frac{A_1h^2}{12R_2}\frac{\partial T_1}{\partial \varphi} = 0;$$

$$\frac{12A_1A_2}{h^2R_2}M_1 + \frac{12A_1A_2}{h^2R_1}M_2 + F_1\frac{\partial T_2}{\partial z} + F_2T_2 + F_3T_1 + F_4\frac{\partial S}{\partial \varphi} + F_5S -$$

$$-\frac{1}{A_1}\frac{\partial A_2}{\partial z}\frac{\partial T_1}{\partial z} + F_6\frac{\partial S}{\partial z} + \frac{A_2}{A_1}\frac{\partial^2 S}{\partial z^2} - 2\frac{\partial^2 S}{\partial \varphi \partial z} + F_7\frac{\partial T_1}{\partial \varphi} + \frac{A_1}{A_2}\frac{\partial^2 T_1}{\partial \varphi^2} -$$

$$-\frac{1}{A_2}\frac{\partial A_1}{\partial \varphi}\frac{\partial T_2}{\partial \varphi} = 0,$$

$$\rho\sigma_s^2h^4 - (\rho-1)\sigma_s(T_1+T_2)h^3 - \left(T_1^2 - T_1T_2 + T_2^2 + 3S^2\right)h^2 -$$

$$-16\left(M_1^2 - M_1M_2 + M_2^2 + 3H^2\right) = 0.$$

From the first eight equations of the system of decision equations (1), we define the internal forces and momenti $T_1, T_2, S, M_1, M_2, H, N_1, N_2$, what are subject to definite solution. The optimal shell thicknesses *h*, which ensures the approximation of the both shell surfaces to the equal strength, is determined from equation nine. We can use the condition of strength instead of the latter, which ensures a precise instant transition of one of the facets of the dam into the end position.

In the system of the decision equations, A_1 and A_2 are Lamé parameters on the median surface of the dam, whose analytical expressions for arch dams with any geometry are given in the monograph [2]. R_1 and R_2 are the main curvical radii; q_1, q_2 and q_3 are the projections of the external forces acting on the unit area of the median surface of the dam (Fig. 2).

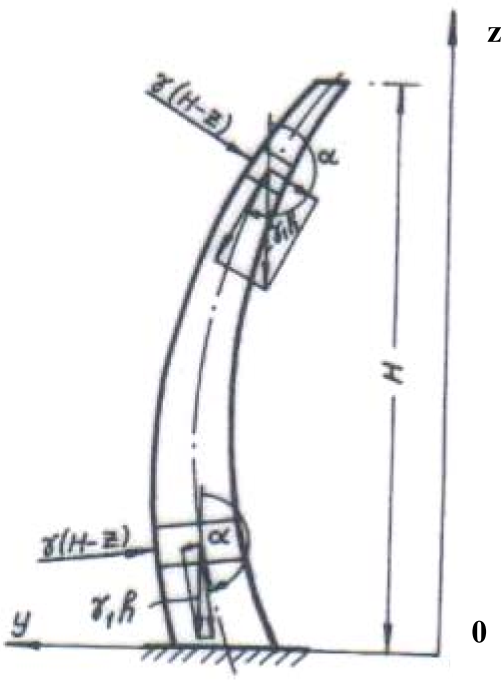

Fig. 2. A plan to define the projection of external forces

$$q_1 = \gamma_1 h \cos\alpha; \quad q_2 = 0; \; q_3 = -\gamma\left(H - Z\right) + \gamma_1 h \sin\alpha, \tag{2}$$

where $$\alpha = \pi - \mathrm{arctg}\frac{dy}{dz}; \; \frac{dy}{dz} = \frac{z - \Delta z}{\sqrt{R_1^2 - \left(z - \Delta z\right)^2}}; \tag{3}$$

γ **and** γ_1 **are the specific weights of the water and dam material (concrete);** F_i $(i = 1....7)$ **are the coefficients with their values equalling to:**

$$F_1=\frac{2}{A_1}\frac{\partial A_2}{\partial z}-\frac{A_2}{A_1^2}\frac{\partial A_1}{\partial z}-\frac{A_2}{A_1h^2}\frac{\partial h^2}{\partial z},$$

$$F_2=\frac{A_2}{A_1^2h}\frac{\partial A_1}{\partial z}\frac{\partial h}{\partial z}-\frac{1}{A_1^2}\frac{\partial A_1}{\partial z}\frac{\partial A_2}{\partial z}-\frac{2}{A_1h}\frac{\partial A_2}{\partial z}\frac{\partial h}{\partial z}+\frac{A_2}{A_1h^3}\frac{\partial h^2}{\partial z}\frac{\partial h}{\partial z}-$$

$$-\frac{A_2}{A_1h}\frac{\partial^2 h}{\partial z^2}+\frac{1}{A_1}\frac{\partial^2 A_2}{\partial z^2}+\frac{1}{A_2^2}\frac{\partial A_2}{\partial \varphi}\frac{\partial A_1}{\partial \varphi}-\frac{1}{A_2}\frac{\partial^2 A_1}{\partial \varphi^2}+\frac{1}{A_2h}\frac{\partial h}{\partial \varphi}\frac{\partial A_1}{\partial \varphi},$$

$$F_3=\frac{1}{A_1^2}\frac{\partial A_1}{\partial z}\frac{\partial A_2}{\partial z}+\frac{1}{A_1h}\frac{\partial h}{\partial z}\frac{\partial A_2}{\partial z}-\frac{1}{A_1}\frac{\partial^2 A_2}{\partial z^2}+\frac{A_1}{A_2^2h}\frac{\partial A_2}{\partial \varphi}\frac{\partial h}{\partial \varphi}-$$

$$-\frac{1}{A_2^2}\frac{\partial A_2}{\partial \varphi}\frac{\partial A_1}{\partial \varphi}-\frac{2}{A_2h}\frac{\partial A_1}{\partial \varphi}\frac{\partial h}{\partial \varphi}+\frac{A_1}{A_2h^3}\frac{\partial h^2}{\partial \varphi}\frac{\partial h}{\partial \varphi}-\frac{A_1}{A_2h}\frac{\partial^2 h}{\partial \varphi^2}+\frac{1}{A_2}\frac{\partial^2 A_1}{\partial \varphi^2}, \qquad (4)$$

$$F_4=\frac{1}{h^2}\frac{\partial h^2}{\partial z}-\frac{2}{A_2}\frac{\partial A_2}{\partial z},$$

$$F_5=\frac{2}{A_1^2}\frac{\partial A_1}{\partial z}\frac{\partial A_1}{\partial \varphi}-\frac{1}{h^3}\frac{\partial h^2}{\partial z}\frac{\partial h}{\partial \varphi}+\frac{2}{h}\frac{\partial^2 h}{\partial z\partial \varphi}-\frac{2}{A_1}\frac{\partial^2 A_1}{\partial z\partial \varphi}+\frac{2}{A_1h}\frac{\partial A_1}{\partial \varphi}\frac{\partial h}{\partial z}+$$

$$+\frac{2}{A_2^2}\frac{\partial A_2}{\partial \varphi}\frac{\partial A_2}{\partial z}-\frac{1}{h^3}\frac{\partial h^2}{\partial \varphi}\frac{\partial h}{\partial z}-\frac{2}{A_2}\frac{\partial^2 A_2}{\partial z\partial \varphi}+\frac{2}{A_2h}\frac{\partial A_2}{\partial z}\frac{\partial h}{\partial \varphi},$$

$$F_6=\frac{1}{h^2}\frac{\partial h^2}{\partial \varphi}-\frac{2}{A_1}\frac{\partial A_1}{\partial \varphi},$$

$$F_7=\frac{2}{A_2}\frac{\partial A_1}{\partial \varphi}-\frac{A_1}{A_2^2}\frac{\partial A_2}{\partial \varphi}-\frac{A_1}{A_2h^2}\frac{\partial h^2}{\partial \varphi}.$$

Statically definable system of decision equations (1) can be solved in terms of internal forces and momenti what excludes any impact of the assumptions of a kinematic nature in the gained results. On the other hand, the solution to such systems in an analytical way is in fact impossible. Therefore, we have concluded that in order to study the deflected mode of the dam body, or solve the system of decision differential equations we must use numerical, in particular, the finite element method [1,4], and to use a numerical method only as finite elements in the base ground and accordingly, in the contact surface “dam-base”.

In order to write the system of decision equations in the finite differences, we make the approximation of the dam study area by means of a finite-difference net [1,3], whose axes are parallel to the coordinate axes Z and φ (See Fig. 3). The net spacing is constant in a vertical direction and equals to *a*, and the spacing b_j is variable in a horizontal direction and is selected by the principle of the net node fitting the contour of the study area [5]. The system of decision equations in the finite diferences is encripted for each net node, which together with the boundary values defined by the method of finite elements makes a mathameticaally correct problem. The particular derivatives of such a net in the central finite differences will be as follows:

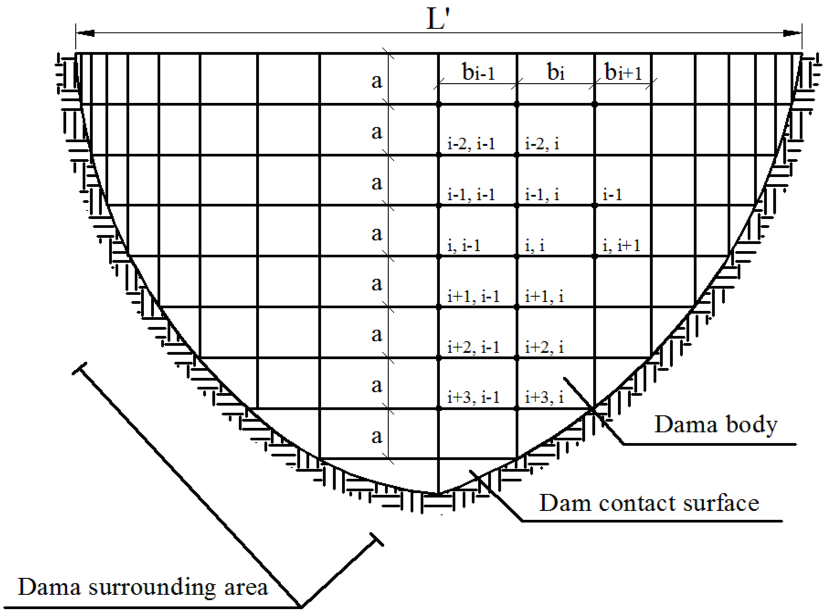

Fig. 3. Approximation of the open area of an arch dam with the net of finite difefrences

$$
\begin{aligned}
&\frac{\partial F}{\partial z}=\frac{1}{2a}\left(F_{i+1,j}-F_{i-1,j}\right);\\
&\frac{\partial F}{\partial \varphi}=\frac{P_j}{a\ell_j\ell_{j-1}}\left[F_{i,j+1}\ell_j^2-F_{i,j-1}\ell_{i-1}^2+F_{i,j}\left(\ell_{j-1}^2-\ell_j^2\right)\right];\\
&\frac{\partial^2 F}{\partial z^2}=\frac{1}{a^2}\left(F_{i+1,j}-2F_{i,j}+F_{i-1,j}\right);\\
&\frac{\partial^2 F}{\partial \varphi^2}=\frac{2P_j}{a^2}\left[F_{i,j+1}\ell_j+F_{i,j-1}\ell_{j-1}-F_{i,j}\left(\ell_j+\ell_{j-1}\right)\right];\\
&\frac{\partial^2 F}{\partial z\partial \varphi}=\frac{P_j}{2a^2\ell_{j-1}\ell_j}\left[\left(F_{i+1,j+1}-F_{i-1,j+1}\right)\ell_j^2+\left(F_{i-1,j-1}-F_{i+1,j-1}\right)\ell_{j-1}^2+\left(F_{i+1,j}-F_{i-1,j}\right)\right]\left(\ell_{j-1}^2-\ell_j^2\right),
\end{aligned}
\tag{6}
$$

where

$$\ell_j=\frac{a}{\varphi_{j+1}-\varphi_j};\ \ell_{j-1}=\frac{a}{\varphi_j-\varphi_{j-1}};\ \ P_j=\frac{a}{\varphi_{j+1}-\varphi_{j-1}}. \tag{7}$$

here *a* is the net spacing along *z* axis; b_j is the net spacing along φ axis; i is the point index along *z* axis; j is the point index along φ axis.

A system of decision equations may be encrypted quite simply by using expression (6), whose further solution gives the optimal parameters of the dam.

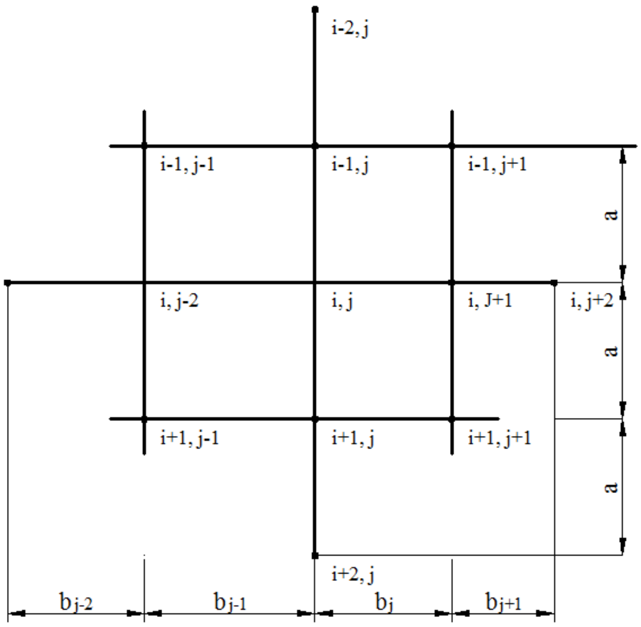

Fig. 4. Net of finite differences

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